

The Scottish Mathematical Council: Mathematical Challenge 2013-14

Solutions to Junior Division: Problems 1

J1.1

A sculpture consists of three large cubes stacked one on top of another without overhanging.

The largest cube has edge 3 metres and its base is on the ground. The next cube has edge 2 metres and the top cube has edge 1 metre.

The exposed surface is to be painted blue. Each tin of paint will cover ten square metres. How many tins of paint will be needed?

Solution

The exposed top surfaces of the smaller cubes equal the covered area of the top surface of the largest cube.

So total of the horizontal areas to be painted is $3 \times 3 = 9 \text{ m}^2$.

Total area of the vertical surfaces is $4 \times (3 \times 3 + 2 \times 2 + 1 \times 1) = 56 \text{ m}^2$.

So the total area is $9 + 56 = 65 \text{ m}^2$.

Hence 7 tins of paint needed (with a half tin left over).

J1.2

Louise's house has a staircase with twelve steps. She can go down either one step or two steps at a time. For example, she could go down

1, 1, 2, 2, 1, 2, 1, 1, 1

In how many different ways can she go down the staircase taking 1 step or 2 steps each time?

Solution

For a staircase with 1 step there is 1 way.

For a staircase with 2 steps there are 2 ways: (1,1) and (2).

For a staircase with 3 steps, Louise must start with either 1 step, followed by 2 ways of completing the remaining 2 steps or 2 steps, followed by 1 way of completing the remaining 1 step.
i.e. 3 ways.

...

For a staircase with n steps, Louise must start with either 1 step, followed by the number of ways of completing the remaining $n - 1$ steps or 2 steps, followed by the number of ways of completing the remaining $n - 2$ steps
i.e. the sum of the previous two staircases.

So the sequence is 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233.

So for a staircase with 12 steps there are 233 possible ways.

J1.3

A corner deli stocks 4 kinds of bread, 5 kinds of meat and 3 kinds of cheese. It sells three types of sandwiches, each made from one kind of bread with either one kind of meat or one kind of cheese or a slice of meat and a slice of cheese as filling.

How many different sandwiches are on sale?

Solution

one bread and one meat: $4 \times 5 = 20$ ways

one bread and one cheese: $4 \times 3 = 12$ ways

one bread, one meat and one cheese: $4 \times 5 \times 3 = 60$ ways

Thus the total number of combinations is 92.

J1.4

In a recent election, six candidates stood and a total of 51880 votes were cast. The winning candidate beat the others by 1336, 7085, 15333, 15654 and 17102 votes respectively.

Candidates lose their deposit if they fail to get more than 5% of the total number of votes cast.

How many candidates lost their deposits?

Solution

Let v be the number of votes obtained by the winner. So the total number of votes cast was

$$v + (v - 1336) + (v - 7085) + (v - 15333) + (v - 15654) + (v - 17102) = 6v - 56510.$$

The total number of votes cast was 51880. Hence

$$6v - 56510 = 51880$$

i.e. $6v = 56510 + 51880 = 108\,390$

i.e. $v = 18065$.

So the six candidates got the following numbers of votes:

$$18065$$

$$18065 - 1336 = 16729$$

$$18065 - 7085 = 10980$$

$$18065 - 15333 = 2732$$

$$18065 - 15654 = 2411$$

$$18065 - 17102 = 963.$$

As 5% of the total is 2594, two candidates each lost their deposit.

J1.5

In a tennis tournament, each match is played between two players, and the winner proceeds to the next round whereas the loser is eliminated. There are no draws.

If necessary, in the first round only, a number of players do not participate.

(a) A particular tournament starts with 256 players and proceeds until there is one overall winner. How many matches are played in this tournament?

(b) If the tournament starts with 296 players and proceeds until there is one overall winner, how many matches are played in this tournament?

Solution

(a) Starting with 256 players, if all play in the 128 first round matches, the number of matches in successive rounds will be 64, 32, 16, 8, 4, 2, 1.

$$\text{Number of matches played} = 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = 255 \text{ matches}$$

(b) In this case there are 40 more players than in (a) so there must be 40 more matches. Thus there are 295 matches.

Alternative Solution

(a) Each match eliminates one player.

So to eliminate 255 players 255 matches are required.

(b) 295 matches